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The coherent time evolution of two coupled quantum dots in a two-mode cavity

X.Z. Yuan[1,a], K.D. Zhu[1], and W.S. Li[2]

[1] Institute of Quantum Optics and Quantum Information, Department of Physics, Shanghai Jiao Tong University, Shanghai 200240, P.R. China
[2] Department of Electronic and Information Engineering, The Hong Kong Polytechnic University, Hong Kong, P.R. China

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Abstract. For two coupled identical quantum dots in a two-mode cavity, we determine the conditions of two-photon and single-photon resonance. It is shown that the exciton-phonon interaction reduces the Rabi frequencies of each model and the Förster interaction between double quantum dots even at absolute zero temperature. The exciton-phonon interaction also makes a contribution to the static exciton-exciton dipole interaction energy. Furthermore, the additional interactions can modify the conditions of photon resonance significantly. A more realistic case of two nonidentical quantum dots is also considered. The influence of parameter misfits on the quantum system is discussed.

PACS. 73.21.La Quantum dots – 71.35.-y Excitons and related phenomena – 42.50.Pq Cavity quantum electrodynamics; micromasers

1 Introduction

Quantum information, quantum computation, and quantum teleportation are becoming more and more promising with the developments of science and technology. So scientists make great efforts to find some physical systems to test and realize their ideas. A variety of hopeful candidates are cavity QED with atoms [1–6], trapped ions [7,8], NMR [9,10], and superconducting devices [11–14]. However, recent developments in semiconductor nanotechnology indicate that excitons in quantum dots have many advantages for the implementation of quantum computing processes [15–17]. To make the results more fruitful, the cavity QED techniques can be used [18–20]. For the system with more than one quantum dot, the coupling among quantum dots becomes important [21]. Therefore, studying the coherent time evolution of the coupled quantum dots in cavity is becoming an important fundamental work in this regime. There are two kinds of interaction among them. One is the static exciton-exciton dipole coupling which exists only when both dots are excited. This interaction can be used to produce entangled few-exciton states via ultrafast laser-pulse sequences [22,23]. The other is Förster interaction which transfers an exciton from one quantum dot to the other [16,24,25]. This kind of interaction is essential to generate maximally entangled Bell states and GHZ states [26] and to implement quantum teleportation [27]. However, the fundamental limitation to the quantum computation based on quantum dot cavity QED is the exciton-phonon interaction which causes decoherences of quantum systems. Therefore, analyzing such effects and finding ways to suppress them are becoming one of the hottest research subjects. The work by Yi et al. [28] investigates two coupled identical quantum dots interacting with the laser pulse. The exciton-phonon interaction is taken into consideration at last, but the authors apply perturbative method. Although reference [19] considers the effects of exciton-phonon interaction more thoroughly, the quantum dot and cavity mode are single.

The main parts of this paper deal with two coupled identical quantum dots which are embedded in a high-Q two-mode cavity and coupled to a common phonon bath and two coupled nonidentical quantum dots embedded in a single mode cavity and a common phonon bath. We get the coherent time evolution of the quantum system and investigate the effects of exciton-phonon interaction with canonical transformation method.

[a] e-mail: yxz@sjtu.edu.cn

2 Model Hamiltonian

In the model mentioned above, each quantum dot has the ground state $|-\rangle$ (no exciton) and first excited state $|+\rangle$ (one exciton). Then the Hamiltonian of the system is given by ($\hbar = 1$) [19,29]

$$H = \omega_1 a_1^+ a_1 + \omega_2 a_2^+ a_2 + \omega \left[S_z^{(1)} + 1/2\right] + \omega \left[S_z^{(2)} + 1/2\right]$$
$$+ 2J_z \left[S_z^{(1)} + 1/2\right] \left[S_z^{(2)} + 1/2\right] + g_1 \left[a_1^+ S_-^{(1)} + a_1 S_+^{(1)}\right]$$
$$+ g_1 \left[a_1^+ S_-^{(2)} + a_1 S_+^{(2)}\right] + g_2 \left[a_2^+ S_-^{(1)} + a_2 S_+^{(1)}\right]$$
$$+ g_2 \left[a_2^+ S_-^{(2)} + a_2 S_+^{(2)}\right] + V \left[S_+^{(1)} S_-^{(2)} + S_-^{(1)} S_+^{(2)}\right]$$
$$+ \sum_{\mathbf{k}} \omega_{\mathbf{k}} b_{\mathbf{k}}^+ b_{\mathbf{k}} + \left[S_z^{(1)} + 1/2\right] \sum_{\mathbf{k}} \left[M_{\mathbf{k}}^{(1)} b_{\mathbf{k}}^+ + M_{\mathbf{k}}^{*(1)} b_{\mathbf{k}}\right]$$
$$+ \left[S_z^{(2)} + 1/2\right] \sum_{\mathbf{k}} \left[M_{\mathbf{k}}^{(2)} b_{\mathbf{k}}^+ + M_{\mathbf{k}}^{*(2)} b_{\mathbf{k}}\right], \quad (1)$$

where $S_+^{(i)} = (|+\rangle\langle-|)_i$, $S_-^{(i)} = (|-\rangle\langle+|)_i$, and $S_z^{(i)} = \frac{1}{2}(|+\rangle\langle+| - |-\rangle\langle-|)_i$ $(i = 1, 2)$, here i denotes the ith quantum dot. ω is the exciton frequency in each of the quantum dot. g_1 (g_2) is the first (second) mode Rabi frequency associated with the exciton-cavity photon interaction. a_1^+ and a_1 (a_2^+ and a_2) are, respectively, the creation and annihilation operators of the first (second) cavity field with frequency ω_1 (ω_2). $b_{\mathbf{k}}^+$ and $b_{\mathbf{k}}$ are those for the phonon with moment $\mathbf{k}$ and frequency $\omega_{\mathbf{k}}$. V represents the Förster interaction which transfers an exciton from one quantum dot to the other. $2J_z$ represents the static exciton-exciton dipole interaction energy. The interaction matrix element $M_{\mathbf{k}}^{(i)}$ is given by [30]

$$M_{\mathbf{k}}^{(i)} = \langle \mathbf{R}_0^{(i)} | w_e(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}_e^{(i)}} - w_h(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}_h^{(i)}} | \mathbf{R}_0^{(i)} \rangle, \quad (2)$$

where $\mathbf{r}_e^{(i)}$ and $\mathbf{r}_h^{(i)}$ are the coordinates of the electron and hole in the ith quantum dot. $|\mathbf{R}_0^{(i)}\rangle$ is the correspondent excitonic state wave function which depends on the structure of the quantum dots and the internal or external electric field [23]. Also, $w_{e,h}(\mathbf{k})$ depends on the type of the exciton-phonon interaction. In what follows, we assume the excitonic state wave functions which localize around the center of each quantum dot are described by the same profile [31].

Applying a canonical transformation with the generator

$$A = \left[S_z^{(1)} + 1/2\right] \sum_{\mathbf{k}} \frac{1}{\omega_{\mathbf{k}}} \left[M_{\mathbf{k}}^{(1)} b_{\mathbf{k}}^+ - M_{\mathbf{k}}^{*(1)} b_{\mathbf{k}}\right]$$
$$+ \left[S_z^{(2)} + 1/2\right] \sum_{\mathbf{k}} \frac{1}{\omega_{\mathbf{k}}} \left[M_{\mathbf{k}}^{(2)} b_{\mathbf{k}}^+ - M_{\mathbf{k}}^{*(2)} b_{\mathbf{k}}\right], \quad (3)$$

we have

$$H' = e^A H e^{-A}$$
$$= (\omega - \Delta) \left[S_z^{(1)} + 1/2\right] + (\omega - \Delta) \left[S_z^{(2)} + 1/2\right]$$
$$+ 2(\Delta_{12} + J_z) \left[S_z^{(1)} + 1/2\right] \left[S_z^{(2)} + 1/2\right]$$
$$+ \omega_1 a_1^+ a_1 + \omega_2 a_2^+ a_2 + \sum_{\mathbf{k}} \omega_{\mathbf{k}} b_{\mathbf{k}}^+ b_{\mathbf{k}}$$
$$+ g_1 \left[a_1^+ S_-^{(1)} e^{-X^{(1)}} + a_1 S_+^{(1)} e^{X^{(1)}}\right]$$
$$+ g_1 \left[a_1^+ S_-^{(2)} e^{-X^{(2)}} + a_1 S_+^{(2)} e^{X^{(2)}}\right]$$
$$+ g_2 \left[a_2^+ S_-^{(1)} e^{-X^{(1)}} + a_2 S_+^{(1)} e^{X^{(1)}}\right]$$
$$+ g_2 \left[a_2^+ S_-^{(2)} e^{-X^{(2)}} + a_2 S_+^{(2)} e^{X^{(2)}}\right]$$
$$+ V \left[S_+^{(1)} S_-^{(2)} e^{X^{(1)} - X^{(2)}} + S_-^{(1)} S_+^{(2)} e^{X^{(2)} - X^{(1)}}\right], \quad (4)$$

where

$$\Delta = \Delta_i = \sum_{\mathbf{k}} \frac{|M_{\mathbf{k}}^{(i)}|^2}{\omega_{\mathbf{k}}}, \quad (5)$$

$$\Delta_{12} = -\sum_{\mathbf{k}} \frac{M_{\mathbf{k}}^{(1)} M_{\mathbf{k}}^{*(2)} + M_{\mathbf{k}}^{*(1)} M_{\mathbf{k}}^{(2)}}{2\omega_{\mathbf{k}}}$$
$$= -\sum_{\mathbf{k}} \frac{|M_{\mathbf{k}}^{(i)}|^2}{\omega_{\mathbf{k}}} \cos\left[\mathbf{k} \cdot (\mathbf{R}_2 - \mathbf{R}_1)\right], \quad (6)$$

$$X^{(i)} = \sum_{\mathbf{k}} \frac{M_{\mathbf{k}}^{(i)} b_{\mathbf{k}}^+ - M_{\mathbf{k}}^{*(i)} b_{\mathbf{k}}}{\omega_{\mathbf{k}}}. \quad (7)$$

Δ_i is the self-energy of the exciton in the ith quantum dot. $2\Delta_{12}$ is the exciton-exciton interaction energy arising from the exciton-phonon interaction and $\mathbf{R}_2 - \mathbf{R}_1$ represents the relative position vector between the center of the two quantum dots.

The nondiagonal transitions exist at finite temperature, but decrease with the decrease of the temperature [30,32]. For quantum dots where the energy separation is greater than 20 meV when the temperature is low enough ($T < 50$ K), the nondiagonal transitions can be neglected [19,33] and phonon states approximately satisfy the Boltzmann distribution. Therefore it is reasonable to just consider diagonal transitions and assume the phonon states in the vacuum state $|0\rangle$ at zero temperature [29]. On the other hand, as the result of quantum fluctuations, some phonon states may change to $|n_{\mathbf{k}}\rangle$ ($n_{\mathbf{k}} \neq 0$) even at zero temperature. Then there exists nondiagonal transition, such as $\langle n_{\mathbf{k}} | H' | 0 \rangle$. But it contains factor $(M_{\mathbf{k}}^{(i)}/\omega_{\mathbf{k}})^{n_{\mathbf{k}}}$ which is small for materials with small λ. $\lambda = \sum_{\mathbf{k}} |M_{\mathbf{k}}^{(i)}|^2/\omega_{\mathbf{k}}^2$ is the Huang-Rhys factor of the exciton in each of the quantum dot. For self-organized

InAs/GaAs quantum dots $\lambda \approx 0.015$ [34] and for other semiconductor quantum dots such as GaAs [35] and InGaAs [15] λ is even more small. Under the conditions mentioned above, it is safe to average H' over the vacuum state to get an effective Hamiltonian

$$\begin{aligned} H_{eff} &= \langle 0|H'|0\rangle \\ &= (\omega - \Delta)\left[S_z^{(1)} + 1/2\right] + (\omega - \Delta)\left[S_z^{(2)} + 1/2\right] \\ &\quad + 2(\Delta_{12} + J_z)\left[S_z^{(1)} + 1/2\right]\left[S_z^{(2)} + 1/2\right] \\ &\quad + \omega_1 a_1^+ a_1 + \omega_2 a_2^+ a_2 + g_1 e^{-\lambda/2}\left[a_1^+ S_-^{(1)} + a_1 S_+^{(1)}\right] \\ &\quad + g_1 e^{-\lambda/2}\left[a_1^+ S_-^{(2)} + a_1 S_+^{(2)}\right] \\ &\quad + g_2 e^{-\lambda/2}\left[a_2^+ S_-^{(1)} + a_2 S_+^{(1)}\right] \\ &\quad + g_2 e^{-\lambda/2}\left[a_2^+ S_-^{(2)} + a_2 S_+^{(2)}\right] \\ &\quad + V e^{-\beta/2}\left[S_+^{(1)} S_-^{(2)} + S_-^{(1)} S_+^{(2)}\right], \end{aligned} \tag{8}$$

where

$$\lambda = \lambda_i = \sum_{\mathbf{k}} \frac{|M_{\mathbf{k}}^{(i)}|^2}{\omega_{\mathbf{k}}^2}, \tag{9}$$

$$\beta = \sum_{\mathbf{k}} \frac{|M_{\mathbf{k}}^{(1)} - M_{\mathbf{k}}^{(2)}|^2}{\omega_{\mathbf{k}}^2} = 2(\lambda - \lambda_{12}), \tag{10}$$

with

$$\lambda_{12} = \sum_{\mathbf{k}} \frac{|M_{\mathbf{k}}^{(i)}|^2}{\omega_{\mathbf{k}}^2} \cos\left[\mathbf{k}\cdot(\mathbf{R}_2 - \mathbf{R}_1)\right]. \tag{11}$$

λ is the Huang-Rhys factor mentioned before. β is an important factor describing the influences of the exciton-phonon interaction on the transfer of excitons from one quantum state to another. The factor λ_{12} is the coupling constant between two excitons arising from the exciton-phonon interaction. Results show that the exciton-phonon interaction affects our quantum system even at zero temperature.

3 Two-photon and single-photon resonance

As an example of the application of the effective Hamiltonian, we assume that the quantum dots are initially prepared in the excited state and the cavity is in the vacuum state

$$|\psi(0)\rangle = |+,+,0,0\rangle. \tag{12}$$

Then the evolution of the state can be expressed as

$$\begin{aligned} |\psi(t)\rangle = \\ C_1(t)|+,+,0,0\rangle + \frac{1}{\sqrt{2}} C_2(t)(|+,-,1,0\rangle + |-,+,1,0\rangle) \\ + \frac{1}{\sqrt{2}} C_3(t)(|+,-,0,1\rangle + |-,+,0,1\rangle) + C_4(t)|-,-,1,1\rangle \\ + C_5(t)|-,-,2,0\rangle + C_6(t)|-,-,0,2\rangle. \end{aligned} \tag{13}$$

From the Schrödinger equation

$$i\frac{d}{dt}|\psi(t)\rangle = H_{eff}|\psi(t)\rangle, \tag{14}$$

we have

$$i\frac{d}{dt}C_1 = g_{10}\sqrt{2}C_2 + g_{20}\sqrt{2}C_3, \tag{15}$$

$$\begin{aligned} i\frac{d}{dt}C_2 &= -(\delta_1 + \Delta_{12} + J_z - V_0)C_2 + g_{10}\sqrt{2}C_1 \\ &\quad + g_{20}\sqrt{2}C_4 + 2g_{10}C_5, \end{aligned} \tag{16}$$

$$\begin{aligned} i\frac{d}{dt}C_3 &= -(\delta_2 + \Delta_{12} + J_z - V_0)C_3 + g_{20}\sqrt{2}C_1 \\ &\quad + g_{10}\sqrt{2}C_4 + 2g_{20}C_6, \end{aligned} \tag{17}$$

$$i\frac{d}{dt}C_4 = -(\delta_1 + \delta_2)C_4 + g_{20}\sqrt{2}C_2 + g_{10}\sqrt{2}C_3, \tag{18}$$

$$i\frac{d}{dt}C_5 = -2\delta_1 C_5 + 2g_{10}C_2, \tag{19}$$

$$i\frac{d}{dt}C_6 = -2\delta_2 C_6 + 2g_{20}C_3, \tag{20}$$

here $g_{10} = g_1 e^{-\lambda/2}$, $g_{20} = g_2 e^{-\lambda/2}$, $V_0 = V e^{-(\lambda-\lambda_{12})}$, $\delta_1 = \omega - \Delta + \Delta_{12} + J_z - \omega_1$, and $\delta_2 = \omega - \Delta + \Delta_{12} + J_z - \omega_2$. To capture the nature of the two-phonon resonance in the system, we use an approximate analysis [36]. We consider the case $|\delta_1|$, $|\delta_2| \gg g_{10}$, g_{20}, and $|\Delta_{12} + J_z - V_0|$ but $|\delta_1 + \delta_2|$ is small. This means that the detunings of the cavity-fields are much large than the Rabi frequencies and the additional interactions but the system is near two-photon resonance. In such case, C_2, C_3, C_5, and C_6 are fast oscillating variables that can be eliminated. To second order approximation (up to $(g_{10}/\delta_1)^2$), we get the equations of C_1 and C_4

$$\begin{aligned} i\frac{d}{dt}C_1 &= \left(\frac{2g_{10}^2}{\delta_1 + \Delta_{12} + J_z - V_0} + \frac{2g_{20}^2}{\delta_2 + \Delta_{12} + J_z - V_0}\right)C_1 \\ &\quad + \left(\frac{2g_{10}g_{20}}{\delta_1 + \Delta_{12} + J_z - V_0} + \frac{2g_{10}g_{20}}{\delta_2 + \Delta_{12} + J_z - V_0}\right)C_4, \end{aligned} \tag{21}$$

$$\begin{aligned} i\frac{d}{dt}C_4 &= \left(\frac{2g_{10}g_{20}}{\delta_1 + \Delta_{12} + J_z - V_0} + \frac{2g_{10}g_{20}}{\delta_2 + \Delta_{12} + J_z - V_0}\right)C_1 \\ &\quad - \left(\delta_1 + \delta_2 - \frac{2g_{10}^2}{\delta_2 + \Delta_{12} + J_z - V_0} - \frac{2g_{20}^2}{\delta_1 + \Delta_{12} + J_z - V_0}\right)C_4. \end{aligned} \tag{22}$$

From the solution we have

$$|C_4(t)|^2 = \frac{4G^2}{4G^2 + \Omega^2}\sin^2\frac{\sqrt{4G^2 + \Omega^2}t}{2}, \tag{23}$$

where

$$G = \left(\frac{2g_{10}g_{20}}{\delta_1 + \Delta_{12} + J_z - V_0} + \frac{2g_{10}g_{20}}{\delta_2 + \Delta_{12} + J_z - V_0}\right), \tag{24}$$

$$\Omega = \delta_1 + \delta_2 + 2(g_{10}^2 - g_{20}^2)\left(\frac{1}{\delta_1 + \Delta_{12} + J_z - V_0} - \frac{1}{\delta_2 + \Delta_{12} + J_z - V_0}\right). \tag{25}$$

The results are similar to those in reference [36]. But something new is that we use two coupled identical quantum dots to take the place of two uncoupled atoms. The static exciton-exciton dipole coupling, the Förster interaction and most important the exciton-phonon interaction are taken into consideration. We find that the exciton-phonon interaction makes a contribution of $2\Delta_{12}$ to the static exciton-exciton dipole interaction energy. Furthermore, the Rabi frequency g_1, g_2, and Förster interaction V are renormalized to $g_{10} = g_1 e^{-\lambda/2}$, $g_{20} = g_2 e^{-\lambda/2}$, and $V_0 = Ve^{-(\lambda-\lambda_{12})}$ respectively. The two parameters of λ and λ_{12} depend on the exciton-phonon coupling strength, the structure of the quantum system, and the internal or external electric field. From the definition of λ and λ_{12}, we have $\lambda > \lambda_{12}$.

The two-photon resonance occurs at $\Omega = 0$. It is obvious that the additional interactions make a contribution to the shift of the two-photon resonance condition $\delta_1+\delta_2 = 0$. With $g_1 = g_2$ and $\delta_1 + \delta_2 = 0$, if $\Delta_{12} + J_z - V_0 \neq 0$, the two-photon resonance happens. This is quite different from the two-atom system. As the additional interactions affect the cooperative process of the system. The influence of exciton-phonon interaction on the coherent time evolution of the system near two-photon resonance is shown in Figure 1. We see that with the increase of the coupling strength of exciton and phonons, the Rabi frequency decreases. Also the maximum population decreases, as the exciton-phonon interaction modifies the condition of two-photon resonance.

For two atoms spatially separated, they do not have direct interaction. However atoms may be located very near in the cavity, then the dipole-dipole interaction among the atoms cannot be neglected. This interaction will produce additional cooperative processes in the system. In this paper, we deal with two quantum dots which have similarities to two atoms. However in realistic situation, the states in such nanostructures are fragile and are affected by the environments. Also the interactions between each quantum dot are important factors which influence the time evolution of the quantum system. Recently, investigating the effects of such additional couplings is becoming one of the hottest subjects in quantum information processing. Our works lie in this respect which is the main difference between our paper and reference [36].

Next we consider the case $|\delta_2| \gg g_{10}$, g_{20}, $|\Delta_{12} + J_z - V_0|$, and $|\delta_1|$. This means that the detuning of the second cavity-field is much large than the Rabi frequencies, the additional interactions, and the detuning of the first cavity-field. Then the second mode has only a little effect

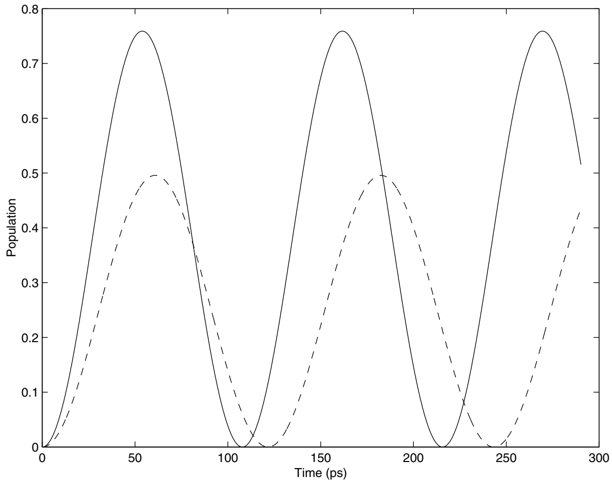

Fig. 1. The coherent time evolution of the population in state $|-,-,1,1\rangle$ near two-photon resonance. $\delta_1 = 11$ meV, $\delta_2 = -11.1$ meV, $g_1 = 1$ meV, $g_2 = 0.81$ meV, $V = 0.5$ meV, $\Delta_{12} + J_z = 1.2$ meV, and $\lambda = 0.05$, $\lambda_{12} = 0.03$ (solid curve), $\lambda = 0.5$, $\lambda_{12} = 0.3$ (dashed curve).

on the behavior of the quantum system. To understand the resonance properties due to the cavity, we first analyze the general feature of the system. Without the two-mode cavity, it is obvious that the eigenenergies of the system are $E_0 = 0$, $E_1 = \omega - \Delta - V_0$, $E_2 = \omega - \Delta + V_0$, and $E_3 = 2(\omega - \Delta) + 2(\Delta_{12} + J_z)$. The corresponding eigenstates are $|\psi_0\rangle = |-,-\rangle$, $|\psi_1\rangle = (|+,-\rangle - |-,+\rangle)/\sqrt{2}$, $|\psi_2\rangle = (|+,-\rangle + |-,+\rangle)/\sqrt{2}$, and $|\psi_3\rangle = |+,+\rangle$. Now we consider the effect of the cavity. With initial state $|\psi(0)\rangle = |+,+,0,0\rangle$, if the cavity frequency $\omega_1 \approx (E_3 - E_0)/2 = \omega - \Delta + \Delta_{12} + J_z$ (i.e., $\delta_1 \approx 0$), the two-photon resonance happens. The system is dominated by the change between states $|+,+,0,0\rangle$ and $|-,-,2,0\rangle$. In such case, the state $(|+,-,1,0\rangle + |-,+,1,0\rangle)/\sqrt{2}$ is suppressed. The two-photon resonance mentioned before occurs in different modes. Now it occurs mainly in the first mode. If the cavity frequency $\omega_1 \approx E_3 - E_2 = \omega - \Delta + 2(\Delta_{12} + J_z) - V_0$, the single-photon resonance happens. The system is dominated by the change between states $|+,+,0,0\rangle$ and $(|+,-,1,0\rangle + |-,+,1,0\rangle)/\sqrt{2}$. At certain time, the double quantum dots are in the Bell state $\psi_{Bell} = (|+,-\rangle + |-,+\rangle)/\sqrt{2}$. Here, the cavity and additional interactions also modify the conditions of the photon resonance.

In such case, C_3, C_4, and C_6 are fast oscillating variables that can be eliminated. Then we get the equations of C_1, C_2, and C_5

$$i\frac{d}{dt}C_1 = \frac{2g_{20}^2}{\delta_2 + \Delta_{12} + J_z - V_0}C_1 + g_{10}\sqrt{2}C_2, \tag{26}$$

$$i\frac{d}{dt}C_2 = -\left(\delta_1 + \Delta_{12} + J_z - V_0 - \frac{2g_{20}^2}{\delta_1 + \delta_2}\right)C_2 + g_{10}\sqrt{2}C_1 + 2g_{10}C_5, \tag{27}$$

$$i\frac{d}{dt}C_5 = -2\delta_1 C_5 + 2g_{10}C_2. \tag{28}$$

Considering the initial condition, the solutions are

$$C_1(t) = \sum_{m=1}^{3} K_m \exp(ix_m t), \tag{29}$$

$$C_2(t) = -\sum_{m=1}^{3} K_m \frac{a+x_m}{b} \exp(ix_m t), \tag{30}$$

$$C_5(t) = \sum_{m=1}^{3} K_m \frac{\sqrt{2}(a+x_m)}{x_m - d} \exp(ix_m t), \tag{31}$$

where

$$K_1 = \frac{(a+x_3)\left(\frac{x_3+a}{x_3-d} - \frac{x_2+a}{x_2-d}\right) - (x_3-x_2)\left(\frac{x_3+a}{x_3-d}\right)}{(x_3-x_1)\left(\frac{x_3+a}{x_3-d} - \frac{x_2+a}{x_2-d}\right) - (x_3-x_2)\left(\frac{x_3+a}{x_3-d} - \frac{x_1+a}{x_1-d}\right)}, \tag{32}$$

$$K_2 = \frac{a + x_3 - K_1(x_3 - x_1)}{x_3 - x_2}, \tag{33}$$

$$K_3 = 1 - K_1 - K_2, \tag{34}$$

with $a = 2g_{20}^2/(\delta_2 + \Delta_{12} + J_z - V_0)$, $b = \sqrt{2}g_{10}$, $c = \delta_1 + \Delta_{12} + J_z - V_0 - 2g_{20}^2/(\delta_1 + \delta_2)$, and $d = 2\delta_1$. Also x_1, x_2, and x_3, are the roots of the equation

$$x^3 + (a - c - d)x^2 + (cd - 3b^2 - ac - ad)x + acd - 2ab^2 + b^2 d = 0. \tag{35}$$

In Figure 2, we plot $|C_2|^2_{max}$ and $|C_5|^2_{max}$ as a function of the cavity frequency ω_1. $\Delta = 0.3$ meV, $V_0 = 0.8$ meV, $\Delta_{12} + J_z = 1.3$ meV, $\omega = 95$ meV, $g_{10} = 0.12$ meV, and $g_{20} = 0.08$ meV. It is shown that the single-photon resonance occurs at the value of $\omega_1 \approx \omega - \Delta + 2(\Delta_{12} + J_z) - V_0 = 96.5$ meV, while the two-photon resonance occurs at the value of $\omega_1 \approx \omega - \Delta + \Delta_{12} + J_z = 96$ meV. The difference of two resonance frequencies is about $\Delta_{12} + J_z - V_0 = 0.5$ meV. The probability of photon emission at resonance is quite high. To some extent, the Bell state is suppressed by the two-photon resonance at $\omega_1 \approx 96$ meV. Therefore if you want to produce the Bell state $\psi_{Bell} = (|+,-\rangle + |-,+\rangle)/\sqrt{2}$, you had better choose $\Delta_{12} + J_z$ and V_0 such that the frequency difference between the single-photon resonance and the two-photon resonance is large enough.

The parameters in Figure 3 are same as those in Figure 2, except $V_0 = 1.3$ meV. Then the single-photon resonance and the two-photon resonance occur at the same frequency $\omega_1 \approx 96$ meV by chance. In such case, the single-photon resonance and the two-photon resonance coexist. The two-photon resonance has higher probability, but it decreases more quickly with the change of cavity frequency ω_1. Due to the influence of two-photon resonance, two side peaks of the single-photon resonance appear.

Through detuning δ_2, this time the second mode has only a little effect on the quantum system. So the two-mode cavity acts more like a single-mode cavity. However taking the additional interactions into account gives us more complicate results than reference [36]. Depending on the values of $\Delta_{12} + J_z$ and V_0, the frequencies of the two-photon resonance and single-photon resonance maybe same or different. Then the quantum system evolves quite differently. This effect should be considered in practical applications. Our results are valid for both resonance and nonresonance cases. Also a recent experimental work [37] has shown that the dipole-dipole interaction affects the two-photon resonance in a similar system.

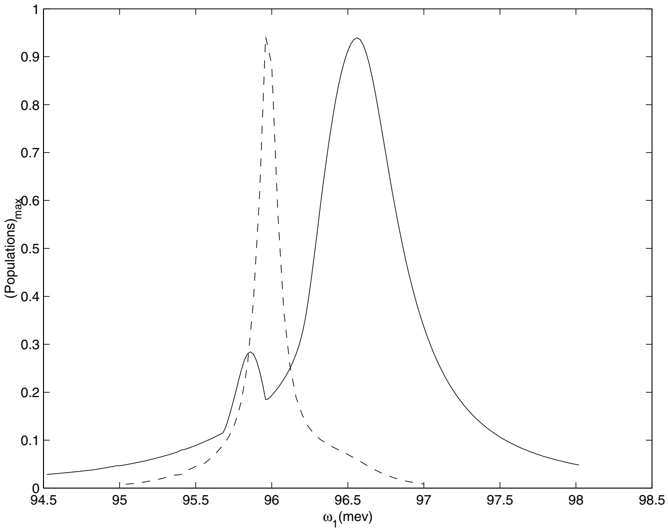

Fig. 2. The maximum value of the populations in state $(|+,-,1,0\rangle + |-,+,1,0\rangle)/\sqrt{2}$ (solid curve), $|-,-,2,0\rangle$ (dashed curve), with respect to the cavity frequency ω_1. $\delta_2 = 30$ meV, $\Delta = 0.3$ meV, $V_0 = 0.8$ meV, $\Delta_{12} + J_z = 1.3$ meV, $\omega = 95$ meV, $g_{10} = 0.12$ meV, and $g_{20} = 0.08$ meV.

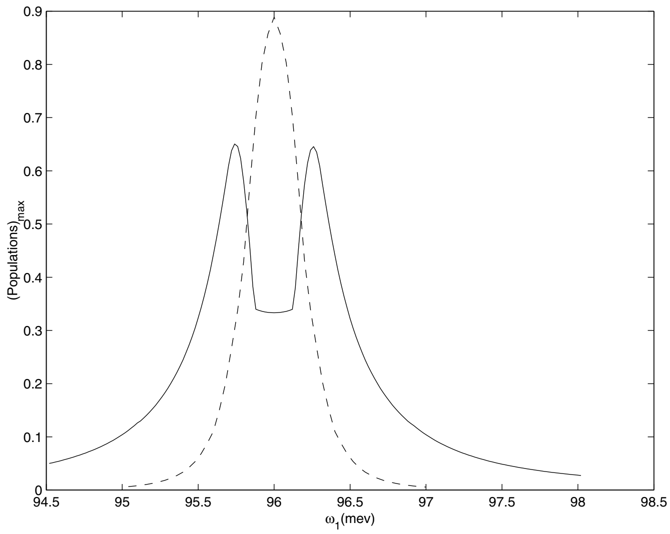

Fig. 3. Same as Figure 2, except $V_0 = 1.3$ meV.

4 Two nonidentical quantum dots

When we use quantum dot systems to take the place of atom systems, something should be consider in more realistic models. First of all, for the quantum computations based on quantum dots, the nanostructures will unavoidably be influenced by the lattice vibrations which we have discussed in this paper. Secondly, we consider only weak inter-dot interaction strengths (about 0.1 meV) which would be expected for two dots with relatively large spacing (about 10 nm) [38]. Therefore, we may neglect inter-dot tunneling of electrons and holes. Finally, different from two atoms, it may well be hard to produce two identical quantum dots in the matrix. Therefore investigating the influence of parameter misfits of two nonidentical quantum dots on the evolution of the system is becoming important. Here for simplicity, we assume the cavity model is single. In the same way we get the effective Hamiltonian of two nonidentical quantum dots in a single mode cavity

$$\begin{aligned} H_{eff} = &(\omega_a - \Delta_1)\left[S_z^{(1)} + 1/2\right] + (\omega_b - \Delta_2)\left[S_z^{(2)} + 1/2\right] \\ &+ 2(\Delta_{12} + J_z)\left[S_z^{(1)} + 1/2\right]\left[S_z^{(2)} + 1/2\right] + \omega_0 a^+ a \\ &+ g_1 e^{-\lambda_1/2}\left[a^+ S_-^{(1)} + a S_+^{(1)}\right] \\ &+ g_2 e^{-\lambda_2/2}\left[a^+ S_-^{(2)} + a S_+^{(2)}\right] \\ &+ V e^{-\beta/2}\left[S_+^{(1)} S_-^{(2)} + S_-^{(1)} S_+^{(2)}\right], \end{aligned} \tag{36}$$

where ω_a (ω_b) is the exciton frequency in the first (second) quantum dot. g_1 (g_2) is the corresponding Rabi frequency. a^+ and a are the creation and annihilation operators of the cavity field with frequency ω_0.

We assume the initial state of the system is

$$|\psi(0)\rangle = |+,+,0\rangle. \tag{37}$$

Then the evolution of the state can be expressed as

$$\begin{aligned} |\psi(t)\rangle = &C_1(t)|+,+,0\rangle + C_2(t)|+,-,1\rangle \\ &+ C_3(t)|-,+,1\rangle + C_4(t)|-,-,2\rangle. \end{aligned} \tag{38}$$

From the Schrödinger equation, we have

$$\begin{aligned} i\frac{d}{dt}C_1 = &[\omega_a - \Delta_1 + \omega_b - \Delta_2 + 2(\Delta_{12} + J_z)]\,C_1 \\ &+ g_{20}C_2 + g_{10}C_3, \end{aligned} \tag{39}$$

$$i\frac{d}{dt}C_2 = (\omega_a - \Delta_1 + \omega_0)C_2 + \sqrt{2}g_{10}C_4 + V_0 C_3 + g_{20}C_1, \tag{40}$$

$$i\frac{d}{dt}C_3 = (\omega_b - \Delta_2 + \omega_0)C_3 + \sqrt{2}g_{20}C_4 + V_0 C_2 + g_{10}C_1, \tag{41}$$

$$i\frac{d}{dt}C_4 = 2\omega_0 C_4 + \sqrt{2}g_{10}C_2 + \sqrt{2}g_{20}C_3. \tag{42}$$

Let $\alpha = \omega_a - \Delta_1 - \omega_b + \Delta_2$, which depends on the size and shape difference of the two quantum dots. Because of loss of symmetry, for two nonidentical quantum dots $\alpha \neq 0$. Here we investigate the influence of α on the evolution of the system in the case of two-photon resonance, i.e., $2\omega_0 = \omega_a - \Delta_1 + \omega_b - \Delta_2 + 2(\Delta_{12} + J_z)$. The solution of the equations gives

$$C_4(t) = -i\sqrt{2}\sum_{n=1}^{2} g_{n0}\sum_{m=1}^{4}\frac{K_m^{(n)}}{x_m}\left[\exp(x_m t) - 1\right], \tag{43}$$

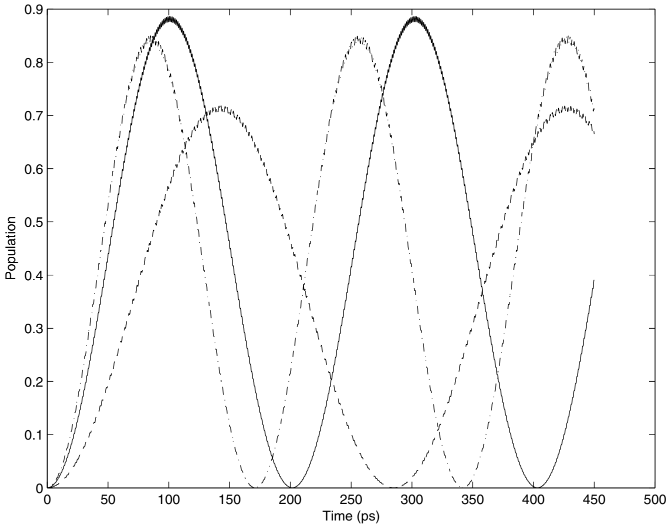

Fig. 4. The coherent time evolution of $|C_4|^2$ near two-photon resonance. $g_{10} = 0.12$ meV, $g_{20} = 0.1$ meV, $\Delta_1 = 0.3$ meV, $\Delta_2 = 0.3$ meV, $V_0 = 0.6$ meV, $\Delta_{12} + J_z = 4.1$ meV, $\omega_b = 80.0$ meV, $\alpha = 0$ meV (solid curve), $\alpha = 3$ meV (dot-dashed curve), and $\alpha = 13$ meV (dashed curve). $\alpha_0 \approx 2\sqrt{(\Delta_{12} + J_z)^2 - V_0^2} = 8.1$ meV.

where x_1, x_2, x_3, and x_4 are the roots of the equation

$$\begin{aligned} &-x^4 + 2i(\Delta_{12} + J_z)x^3 + [(\omega_0 - \omega_a + \Delta_1)(\omega_0 - \omega_b + \Delta_2) \\ &-3g_{10}^2 - 3g_{20}^2 - V_0^2]\,x^2 + i\left[2g_{10}^2(\omega_0 - \omega_b + \Delta_2)\right. \\ &+g_{10}^2(\omega_0 - \omega_a + \Delta_1) + 2g_{20}^2(\omega_0 - \omega_a + \Delta_1) \\ &\left.+g_{20}^2(\omega_0 - \omega_b + \Delta_2) + 6g_{10}g_{20}V_0\right]x - 2g_{10}^4 \\ &-2g_{20}^4 + 4g_{10}^2 g_{20}^2 = 0. \end{aligned} \tag{44}$$

Also $K_1^{(n)}$, $K_2^{(n)}$, $K_3^{(n)}$, and $K_4^{(n)}$ are the roots of the equations

$$K_1^{(n)} + K_2^{(n)} + K_3^{(n)} + K_4^{(n)} = 0, \tag{45}$$

$$\begin{aligned} &(x_2 + x_3 + x_4)K_1^{(n)} + (x_1 + x_3 + x_4)K_2^{(n)} \\ &+(x_1 + x_2 + x_4)K_3^{(n)} + (x_1 + x_2 + x_3)K_4^{(n)} = A_1^{(n)}, \end{aligned} \tag{46}$$

$$\begin{aligned} &(x_2x_3 + x_2x_4 + x_3x_4)K_1^{(n)} + (x_1x_3 + x_1x_4 + x_3x_4)K_2^{(n)} \\ &+(x_1x_2 + x_1x_4 + x_2x_4)K_3^{(n)} \\ &+(x_1x_2 + x_1x_3 + x_2x_3)K_4^{(n)} = A_2^{(n)}, \end{aligned} \tag{47}$$

$$\begin{aligned} &x_2x_3x_4K_1^{(n)} + x_1x_3x_4K_2^{(n)} + x_1x_2x_4K_3^{(n)} \\ &+x_1x_2x_3K_4^{(n)} = A_3^{(n)}, \end{aligned} \tag{48}$$

with $A_1^{(1)} = ig_{20}$, $A_2^{(1)} = -g_{10}V_0 - (\omega_0 - \omega_b + \Delta_2)g_{20}$, $A_3^{(1)} = 2ig_{20}[g_{20}^2 - g_{10}^2]$, $A_1^{(2)} = ig_{10}$, $A_2^{(2)} = -g_{20}V_0 - (\omega_0 - \omega_a + \Delta_1)g_{10}$, and $A_3^{(2)} = 2ig_{10}[g_{10}^2 - g_{20}^2]$.

In Figure 4, we plot $|C_4|^2$ as a function of time for different value of α. It is obvious that two-photon resonance appears while two other states $|+,-,1\rangle$ and $|-,+,1\rangle$ are inhibited. With the increase of α ($\alpha \geq 0$), the Rabi frequency increases first, then it decreases. The turning point is at $\alpha_0 \approx 2\sqrt{(\Delta_{12}+J_z)^2 - V_0^2}$ ($|\Delta_{12}+J_z| > V_0$). For $\alpha(g_{10}-g_{20}) > 0$, just exchanging the value of g_{10} and g_{20}, the probability of two photo emission increases. When $\alpha = \alpha_0$ the evolution of the system is rather complicate, as two-photon resonance and single-photon resonance coexist. Though the condition of two-phonon resonance is satisfied, it is not a good time to realize two-phonon resonance. This can be avoided by designing the quantum system properly.

Before concluding we take the effects of cavity decay and spontaneous emission of the excitons into consideration. In this case, the time evolution of two coupled quantum dots in a two-mode cavity is given by the following master equation

$$\begin{aligned}\frac{\partial\rho}{\partial t} &= -i[H_{eff},\rho] - \frac{1}{2}\kappa_1(a_1^+a_1\rho - 2a_1\rho a_1^+ + \rho a_1^+a_1)\\ &- \frac{1}{2}\kappa_2(a_2^+a_2\rho - 2a_2\rho a_2^+ + \rho a_2^+a_2)\\ &- \frac{1}{2}\sum_{i,j=1}^{2}\Gamma_{ij}(S_i^+S_j^-\rho - 2S_j^-\rho S_i^+ + \rho S_i^+S_j^-), \quad (49)\end{aligned}$$

where κ_1, κ_2 are the cavity decay rates of the first and second mode, whereas Γ_{ij} describe the spontaneous emission rates ($i = j$) and the collective damping of the excitons ($i \neq j$). Solving this equation, we certainly can obtain more reasonable results. Tanaś and Ficek [39] have discussed the creation of entanglement between two two-level atoms in the dissipative process of spontaneous emission and shown that spontaneous emission can lead to a transient entanglement between the atoms. Pathak and Agarwal [36] also considered the effect of cavity decay on two-atom two-photon vacuum Rabi oscillation. If both the losses due to spontaneous emission and cavity decay in the coupled quantum dots within a two-mode cavity are considered simultaneously the calculations will be more complicated. This will be given in due course.

5 Conclusions

We have investigated the coherent time evolution of two coupled identical quantum dots in a two-mode cavity. The exciton-phonon interaction contributes $2\Delta_{12}$ to the static exciton-exciton dipole interaction energy. Its effect also adds a factor of $e^{-\lambda/2}$ on the Rabi frequency of each mode and a factor of $e^{-(\lambda-\lambda_{12})}$ on the Förster interaction V. Furthermore, the conditions of two-photon and single-photon resonance are given. Results show that the additional interactions change the resonance conditions. Different from the two uncoupled atoms, we have shown that for symmetric couplings ($g_1 = g_2$), the two-photon emission also exists. As the additional interactions affect the cooperative process of the system. Finally, we consider a more realistic case, i.e., two nonidentical quantum dots. The parameter misfits of the two quantum dots change the Rabi frequency and influence the evolution of the quantum system. Although decoherence mechanisms due to phonon-exciton interactions are a central ingredient of the present model, we here have only considered static exciton self-energy shifts, induced exciton-exciton interaction and modifications to exciton-cavity and Förster couplings. Real decoherence effects produced by exciton-phonon interactions are more important than changes in the resonance conditions. We are sure that this paper would be much more complete if the performance of the considered coherent evolution could be evaluated as a function of the appropriate strong coupling constants and characterizations. This is a challenging work even for the simpler case such as a single quantum dot with exciton-phonon coupling. Very recently, Förstner et al. [40] have calculated the phonon-assisted damping of Rabi oscillations in a single quantum dot within a density matrix theory, but the exciton-phonon interaction is only considered up to the second order of a correlation expansion. We will follow the method proposed by Förstner et al. [40] to treat the coupled quantum dots in a common phonon bath numerically. This complicated work is underway and will be presented elsewhere. Finally we hope the present study will give help to understand thoroughly the real dynamic behavior of quantum dot systems in an optical microcavity.

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